

# Axial-moment interaction and load path dependency for steel columns

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## ABSTRACT

This paper discusses the effect of out-of-straightness direction, major-minor moment interaction and path dependency for steel columns. This has been undertaken by using the Extended Direct Analysis (EDA) method which has been developed for analysing a simple low-rise 3D steel frame structure under 3D biaxial loadings in arbitrary load-paths from first principles. It takes into account non-ideal conditions, residual stresses, member out-of-straightness, statistical variation in capacities, plasticity, and second-order geometric effects.

It was found steel columns are path dependant, with the plastic capacity reducing by up to 18% when the loading occurs in the major and minor axis at the same time. By completing bi-directional loading on a simple 3D frame it was found that the maximum critical load on the major and minor axes reduced by 10% when compared to the 2D EDA method which does not consider major-minor moment interaction. Therefore it is concluded that the tangent modulus is a function of axial load, moment and load path. Further research is required to account for torsional effects and shear before this method can be used for design.

## 1. INTRODUCTION

In New Zealand and around the world there are several different types of analyses that are used to determine the demands on steel members. First-order analysis considers first-order geometric nonlinear effects while second-order analysis considers second-order geometric nonlinear effects (P- $\delta$  and P- $\Delta$ ). Analyses can also be considered as elastic or plastic. Elastic analysis only considers elastic capacity while plastic analysis considers plastic capacity of the steel members.

Prior to the 1997 New Zealand Steel design code (NZS3404: 1997), the design of steel members in New Zealand was performed using computer programs that implemented first-order elastic analysis. The results were adjusted using correction factors in order to obtain more accurate estimates of the demands due to second order effects.

In current New Zealand practice either first-order elastic moments are amplified in order to account for second-order geometric effects, or a second-order elastic analysis is computed directly. Neither of these methods predict the likely forces or moments on steel frames without the addition of modification factors. These modification factors are required to account for member and frame imperfections as well as potential inelasticity in the steel members. Other issues associated with the design of steel structures in New Zealand include:

- i. The amplification factors used to modify the first-order analyses are based on approximations which were developed for some idealised cases. Realistic structures are more complex than these ideal cases, and thus the amplification factors for complex structures may be different to the factors from the ideal cases.

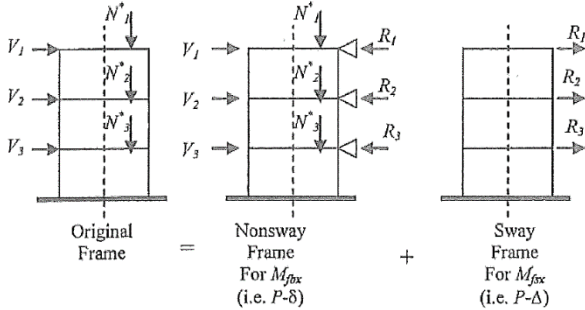
- ii. Due to limited guidance from the New Zealand steel design code (NZS3404: 1997) there has been confusion on how to apply first-order corrections due to multiple methods being available, with no one method being general enough to consider all the different loading cases and structural forms.
- iii. The methods used in NZS3404 emphasise the formulation of computer design moments and forces, but not displacements. Displacements are important because of path dependency and P- $\Delta$  second-order effects which can reduce capacity and increase demands on members.
- iv. Plastic analysis design is seldom used because the New Zealand design code (NZS3404: 1997) has limited guidance on the use of plastic analysis.

### 1.1. NZS3404 Methods

As previously noted, there are several methods available within the New Zealand steel design code. One method which is described within Appendix F of NZS3404 (1997) can be used for elastically responding rectangular frames. The calculation of critical moments using the Appendix F method is completed by splitting the structure into two parts, a braced (No-sway) frame and an unbraced (Sway) frame.

Using computer software, the first-order braced frame moments ( $M_{fb}$ ) are obtained by applying all of the forces on the structural frame while artificially bracing the structure to stop any lateral displacement. The reactions obtained from the artificial bracing are then applied to the sway frame in order to obtain the first-order sway frame moments ( $M_{fs}$ ) as shown below in Figure 1.

Once the moments for braced and sway cases are found the maximum sway magnification factor ( $\delta_s$ ) and the maximum braced magnification factor ( $\delta_b$ ) are applied to obtain the moment demand ( $M^*$ ) as shown in equation (1).



**Figure 1.** Braced and sway frame calculations

$$M^* = \delta_b(\delta_s M_{fs} + M_{fb}) \quad \text{Equation (1)}$$

The Appendix F method performs poorly for complex geometries with no single inter-storey height. In these situations, other approximate methods (including using effective length factors) or a second-order geometric analysis should be used. For an analysis to be performed properly, all important capacity reducing effects need to be considered. These effects include thermal residual stress effects, member out-of-straightness, second-order geometric effects and non-ideal conditions.

## 1.2. Extended Direct Analysis

The Direct Analysis Method has been developed in America by AISC (2005) and is a more thorough method than the current methods outlined in the New Zealand steel code (2004). The Direct Analysis method takes into account frame non-ideal conditions, partial yielding, residual stresses, member out-of-straightness and second-order geometric effects. Partial yielding, residual stresses and member out-of-straightness effects are captured through the use of a stiffness reduction factor that acts to reduce the tangent stiffness of the steel member, thereby decreasing the capacity of the member. The stiffness reduction factor can be obtained by finding the difference between the Euler buckling curve and the AISC (AISC, 2005) column design curve.

Some of the ideas used in the Direct Analysis method can be used in a New Zealand method for direct analysis. The US Direct Analysis method is not directly applicable for NZ or Australian design because the US code employs only one column design curve while the NZ/Australian code employs five different curves. The different column design curves are a result of assuming different residual stress patterns and out-of-straightness on a given member.

In 2009 an undergraduate student from the University of Canterbury proposed a method for back calculating the stiffness reduction factors from the five NZ column design curves and the Euler buckling curve. The result of this research was the development of five different stiffness reduction factor tables that could be used to determine the exact stiffness reduction factor from a given axial force ratio and residual stress distribution.

Lu (2009) went further and proposed an analysis method called Extended Direct Analysis (EDA) which encapsulated all the destabilizing effects considered in the US Direct Analysis method while using the five NZ column design curves.

MASTAN 2 (Ziemian, 2000) was used to implement EDA due to its second-order inelastic analysis capabilities under uni-directional loading. A lookup table of computed stiffness reduction factors was used rather than curves because MASTAN 2 could easily find the exact value through interpolation. Only two additional files were required to modify MASTAN 2 in order to perform EDA. The first file provided a lookup table of the stiffness reduction factors and the second file obtained a stiffness reduction factor (corresponding to the current axial load in each member) from the first file and computed the new stiffness for each load increment.

The current limitation of the EDA method is that the frame must be fully braced out-of-plane to ensure that out-of-plane flexural-lateral-torsional buckling does not affect the analysis. Out-of-plane actions are assumed to be small and a separate set of checks have to be performed to ensure that out-of-plane instabilities do not exist. Due to these restrictions, EDA method developed by Lu (2009) can only be used for monotonic loading and does not consider path dependency or biaxial loading.

There was therefore a need to modify the current EDA method so that it can be used to analyse simple low-rise 3D steel frames under 3D biaxial loadings in arbitrary load-paths. This brought EDA into a three dimensional realm where effects including path dependency and major/minor axis moment interaction can be considered.

The following questions need to be answered in order to establish EDA in three dimensions:

- How can residual stress and out-of-straightness be taken into account when modelling the New Zealand column curves?
- Is path dependency a significant capacity reducing effect in steel columns?
- What are the main differences between current NZ standards and EDA in three dimensions?

It should be noted that beam and column flexural-lateral-torsional buckling effects have been ignored due to their complexity. Table 1 outlines the differences between the described analysis methods and the properties of the proposed EDA method in three dimensions.

## 2. METHOD AND MATERIALS

### 2.1. FEDEASLab Software

In the EDA approach by Lu (2009), the New Zealand column curve is implemented by reducing the stiffness. The factor that is used to reduce the stiffness is called the stiffness reduction factor (SRF). The stiffness reduction factor is a function of axial load and does not consider the effect of moment in the member. This is shown below in Equation (2).

$$E_t I = SRF(N) * E I \quad \text{Equation (2)}$$

Where E is the youngs modulus,  $E_t$  is the tangent modulus, N is the axial force on the member and I is the second moment of area of the section. This implementation means the members can still be modelled as idealized members with EI flexural rigidity.

In 3D EDA, FEDEASLab (Filippou, 2002) was used to create members that are derived from the ground-up based on fibre sections with material stress-strain relationships. Therefore, 3D EDA allowed the changes in tangent stiffness due to both axial force and moment in the member to be analysed. FEDEASLab also has the tools to implement residual stresses and out-of-straightness explicitly and easily.

It was for these reasons that FEDEASLab was chosen to undertake the chosen software for this project.

### 2.2. Non-ideal Conditions

Non-ideal conditions of a steel frame (Including incidental patterned gravity load effects, temperature gradients across the structure, foundation settlement, uneven column shortening, and any other effects that could induce sway that is not explicitly considered in the analysis) can cause instability. These non-ideal conditions are considered in the model through the use of notional loads. Notional loads are taken to be 0.002 times the factored gravity load effects applied horizontally at the top of each storey. It is important to note that the notional loads will be encapsulated in the critical horizontal loads from the analyses and is not explicitly considered.

The statistical variation in capacities was considered using a strength reduction factor applied to both the yield strength and the Youngs modulus. This is shown in equations (3) and (4).

$$E_{input} = \Phi E \quad \text{Equation (3)}$$

$$F_{y,input} = \Phi F_y \quad \text{Equation (4)}$$

$$\Phi = 0.9 \quad \text{Equation (5)}$$

The strength reduction factor,  $\Phi$ , accounts for the capacity of the member being lower than the capacity it was designed for. Applying a strength reduction factor gives more conservative results.

**Table 1.** Comparison of different steel analysis methods

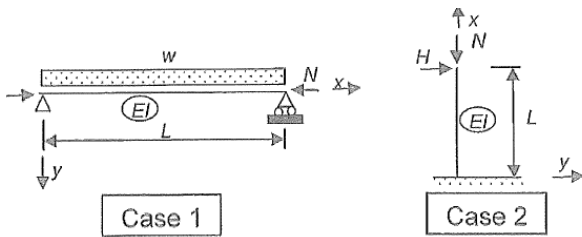
	NZS3404	Second Order Inelastic analysis programmes	Extended direct analysis (EDA) 2D	Extended Direct Analysis (EDA) 3D
<b>Type of Analysis</b>	1st order elastic analysis	2nd order inelastic analysis	2nd order inelastic analysis	2nd order inelastic analysis
<b>Geometric nonlinearity</b>	Considered using magnification factors	considered in the program based on second order geometry options	considered in the program based on second order geometry options	considered in the program based on second order geometry options
<b>Initial out-of-plumbness</b>	Additional notional loads	Additional notional loads	Additional notional loads	Additional notional loads
<b>Initial residual stress</b>	Column curves check each member individually	Column curves check each member individually	Stiffness reduction factor (SRF)	Parabolic residual stress implemented
<b>Out-of-Straightness</b>	Column curves check each member individually	Column curves check each member individually	Stiffness reduction factor (SRF)	Out-of-straightness profile manually implemented
<b>In-plane Check</b>	Manually	Manually	Automatically	Automatically

### 2.3. Second-order Analysis Validation

There had to be full confidence in the software's second-order nonlinear geometric analysis abilities. Therefore, a simple monotonic second-order analysis validation was performed to evaluate the error in the software's second-order geometric nonlinear analysis capabilities.

The second-order analysis validation method was taken from the AISC Steel Construction Manual (AISC, 2005) and compares the computational max moment ( $M_{\max}$ ) and max displacement ( $Y_{\max}$ ) with the theoretical max moment and max displacement. The AISC validation provides two bench mark problems (case 1 and case 2) and requires the computational solutions to be within three percent of the theoretical solutions. These cases can be found in Figure 2 below.

MASTAN2 and FEDEASLab were tested using 2, 4, 6, 8 and 10 element subdivisions. Table 2 shows the results of the validation for the max moment in case 1 and case 2. It can be seen that 6 element subdivisions are required to provide sufficient accuracy for all cases. 8 elements were used to provide midpoint and quarter-point results in FEDEASLab.



**Figure 2.** Second-order validation cases

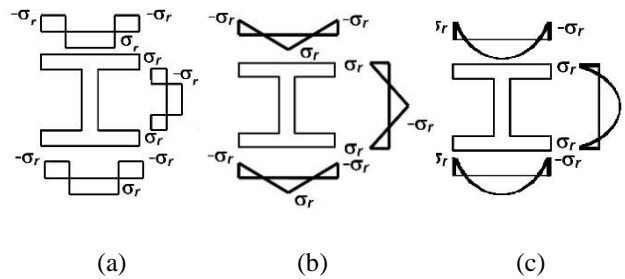
### 2.4. Residual Stress Profile

The type of initial residual stress (IRS) distribution in a steel member is one of the key components for developing the New Zealand column curves.

The magnitude and distribution of IRS in a section not only depends on the type of manufacturing process (For example hot-rolled, welded or cold-formed sections), but also on the type of cross section, thickness of the section, cooling conditions, rolling temperature, straightening method and steel properties. It is time-consuming and impractical to incorporate the exact IRS distribution for all members into the analysis.

Universal Hot-rolled column sections, which are to be used for this analysis, are generally expected to form tension at the centre of the web and at the edge of the flange due to these locations having fast cooling rates. The web-flange junction on the other hand tends to cool at a much slower rate, which allows initial tensile residual stress to occur.

In order to model this IRS profile, three potential residual stress profiles were considered. The profiles were piecewise constant, linear and parabolic profiles, as shown in Figure 3 below.



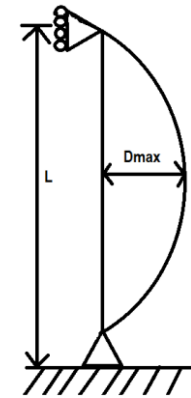
**Figure 3.** Residual stress profiles for (a) Piecewise constant, (b) Linear and (c) Parabolic

**Table 2.** Second-order analysis validation results for case 1 and case 2

Software	Number of Elements				
	2	4	6	8	10
Case 1					
FEDEASLab Mmax% error	12.79	4.05	1.8	0.94	0.53
MASTAN2 Mmax % error	6.63	2.55	1.72	1.4	1.25
FEDEASLab Ymax % error	7.97	1.89	0.62	0.2	0.05
MASTAN2 Ymax% error	4.09	1.89	1.47	1.38	1.3
Case 2					
FEDEASLab Mmax% error	12.79	4.05	1.8	0.94	0.53
MASTAN2 Mmax % error	6.63	2.55	1.72	1.4	1.25
FEDEASLab Ymax % error	21.04	6.67	2.95	1.55	0.88
MASTAN2 Ymax% error	10.54	3.74	2.31	1.81	1.58

## 2.5. Initial Member Out-of-Straightness

The member out-of-straightness is another key component for the development of the New Zealand column curves. Initial out-of-straightness, denoted by  $D_{\max}$  in Figure 4, accounts for any manufacturing out-of-straightness. The maximum out-of-straightness is defined in the model as a proportion of the length of the column being considered. The out-of-straightness profile is modelled using a half sine wave. The maximum out-of-straightness can be said to be equal to the amplitude of the sine function.

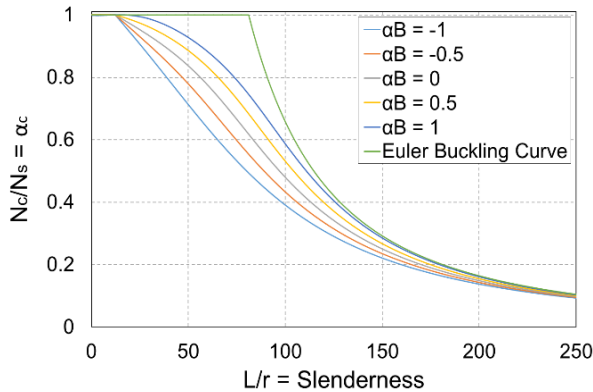


**Figure 4.** Out-of-straightness model

## 3. RESULTS AND DISCUSSION

### 3.1. Column Curve Matching

As stated earlier, there are five current Australia/New Zealand column design curves used to encapsulate the different steel member types and properties. These five curves, as well as the Euler buckling curve, can be seen in Figure 5. The analysis only considers the  $\alpha_B = 0$  column design curve as only hot rolled members are used in the analysis.



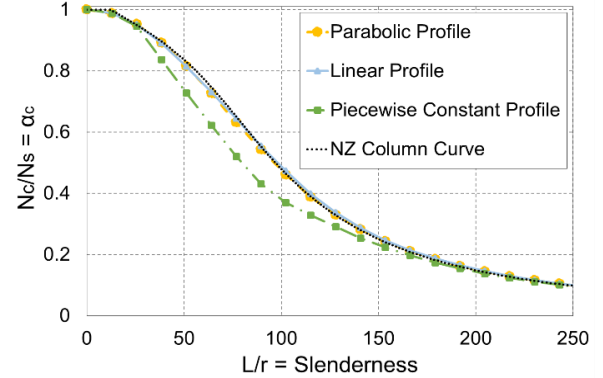
**Figure 5.** The New Zealand column design curves and the Euler buckling curve

A force based loading analysis was undertaken on multiple pin-roller models (Shown in Figure 4) in order to find the residual stress profile, maximum residual stress and maximum out-of-straightness that matched the  $\alpha_B = 0$  New Zealand column curve. The maximum out-of-straightness was varied between 1/1500 and 1/100 of the column length, all three residual profiles were considered and the maximum residual stress profile was varied between  $0.1f_y$  and  $0.5f_y$ .

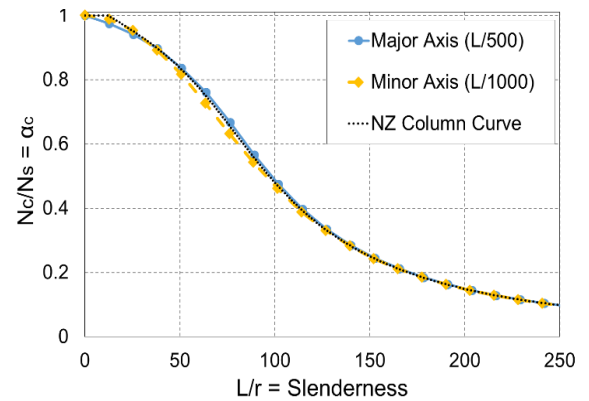
It was found that a parabolic residual stress profile with a maximum residual stress of  $0.3f_y$  best matched the  $\alpha_B = 0$  New Zealand column curve.

The out-of-straightness that best matched the New Zealand column curve was found to be 1/1000 of the column length for the minor axis and 1/500 of the column length for the major axis.

A comparison of the residual stress profiles in the



**Figure 6.** Residual stress profile comparison



**Figure 7.** NZ Column design curve matching

major axis with an out-of-straightness of 1/500 of the column length and a maximum residual stress of  $0.3f_y$  is shown in Figure 6. The final simulated minor and major column curves are shown in Figure 7.

In addition, several different universal column sizes ranging between 150UC30 and 310UC137 were used in the same model and it was found that the model was independent of member size.

In summary, in order to model the New Zealand column curves using EDA for a universal column in FEDEASLab, the parameters outlined in Table 3 will be used.

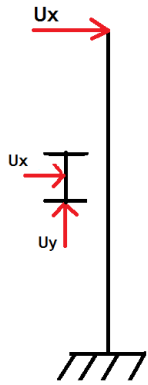
**Table 3.** Summary of EDA parameters

Property	Major Axis	Minor Axis
<i>Out-of-straightness</i>	L/500	L/1000
<i>Residual Stress Profile</i>	Parabolic residual stress profile with $0.3f_y$ as the maximum initial residual stress.	

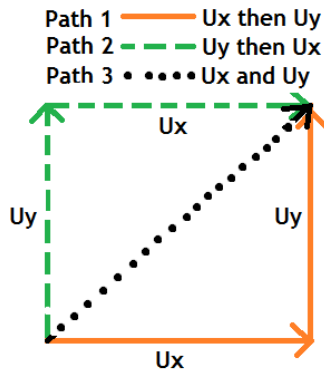
### 3.2. Path Dependency Analysis

Using the out-of-straightness and residual stress profiles shown in Table 3 in order to accurately model the New Zealand column curves, a simple cantilever column model (shown in Figure 8) was generated in FEDEASLab to check for path dependency and out-of-straightness direction effects.

In reality there is no way of telling how the direction of load will be applied. For this reason it was important to determine whether the cantilever column model was path dependant, and if so, what load combination is the most critical. For this reason a path dependency analysis, shown in Figure 9, was completed to check if the column was path dependant for monotonic response at 1% drift.



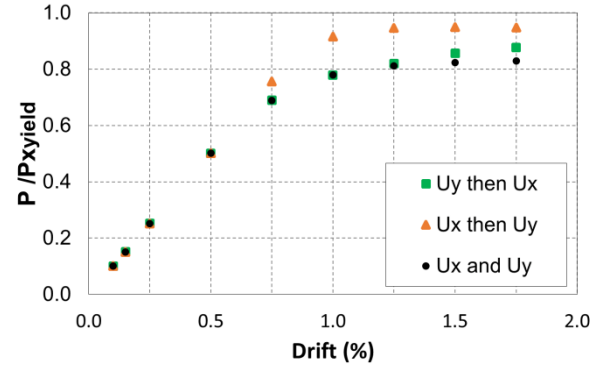
**Figure 8.** Biaxial model



**Figure 9.** Pathways being considered

From the results of this model (seen in Figure 10) it was found that the biaxial model was path dependant, reducing the plastic capacity by up to 18% once yielding occurs at approximately 0.75% drift. It can be seen that the worst case is when the displacement is applied in the major and minor axis simultaneously.

The biaxial model was also used to find what effect the direction of out-of-straightness had on the column. It was found that providing an out-of-straightness opposite to the direction of loading was the worst case. This value was compared to the same model with no out-of-straightness. It was found that the difference in capacities was just under 3%.



**Figure 10.** Monatomic plastic response of column pushed through different pathways

### 3.3. 2D Frame Analysis

EDA of two 2D frames were undertaken using FESEASLab in order to make comparisons between 3D EDA, Lu (2009) 2D EDA and the Appendix F method. The analyses were undertaken using a push over analysis to find the critical horizontal loads for each frame. Both frames had the same section sizes but one frame had the columns orientated for minor axis bending while the other frame had the columns orientated for major axis bending.

Using 3D EDA the 2D frame, with the configuration shown in Figure 11, was subjected to horizontal and vertical loading with out of plane effects restrained. Both frames had out-of-straightness applied to the columns. Comparing the direction of the out-of-straightness, the most critical direction was found to be in the opposite direction of the horizontal loading, which matches the results seen in the biaxial model shown in Figure 8.

An out-of-straightness of 1/1000 of the column length was used for the minor axis bending frame and an out-of-straightness of 1/500 of the column length was used for the major axis bending frame (as determined from the curve matching analysis). A residual stress of  $0.3f_y$  was assigned to all the members of the 2D frame. Vertical and horizontal loading variables are outlined in equations (6), (7), and (8). Results are shown in Table 4.

**Table 4.** 2D frame analysis results

2D Frame Analysis (Summary of Results)				
	Appendix F	Lu (2009) EDA Method	FEDEASLab	
	(Assuming alternative method)	(6 elements per member)	(6 elements per member)	(8 elements per member)
Horizontal Load (Major)	139000 N	150900 N	145000 N	145000 N
Horizontal Load (Minor)	66000 N	75100 N	69500 N	69700 N



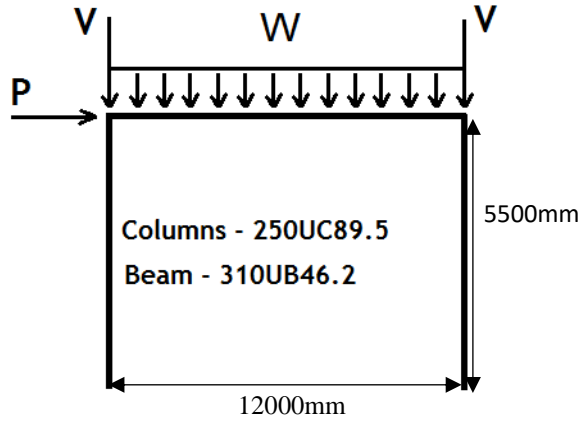


Figure 11. 2D Frame configuration

$$V = 500,000N \quad \text{Equation (6)}$$

$$W = 3.81N/mm \quad \text{Equation (7)}$$

$$P = \text{Push over critical load} \quad \text{Equation (8)}$$

As previously stated, the Appendix F method provides overly conservative results that lead to uneconomical design. This can be seen in Table 4 where the Appendix F method provides the minimum critical load. Lu (2009) 2D EDA method provides less conservative results, varying from the results of the 3D EDA method by 4%. This is because FEDEASLab is able to model the tangent stiffness as a function of axial force and moment on the member.

Lu (2009) 2D EDA is only able to model the tangent stiffness as a function of axial force prior to yielding. It is important to note that path dependency effects are not considered in the 2D frame analysis because a simple monotonic loading regime is considered.

### 3.4. 3D Frame Analysis

EDA of a 3D frame, shown in Figure 12, was undertaken to compare the difference in capacity between the 2D and 3D EDA methods.

A path dependency analysis, (like the one shown in Figure 9) was completed using 3D EDA in order to check that the 3D frame was path dependant for monotonic response at 1% drift. From this analysis it was found that the worst case, just like the bi-axial column, was when the load was applied in the major and minor axis simultaneously.

Using monotonic incremental load analysis, with the increments being a proportion of the total critical load obtained in the major and minor axis 2D frames, it was found that the minor axis failed first.

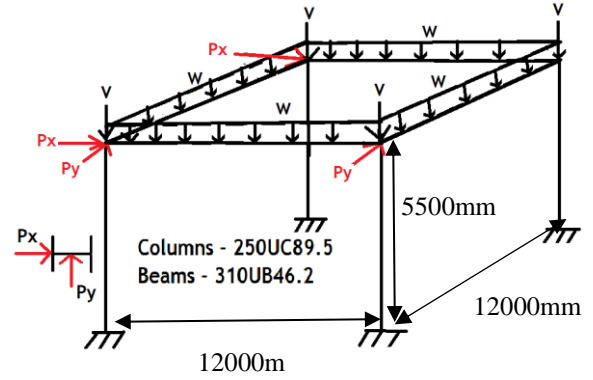


Figure 12. 3D Frame configuration

The same 3D model was implemented in MASTAN2 using Lu (2009) 2D EDA method, with the results shown in Table 5. The difference in capacity between the 3D EDA method and the 2D EDA method was found to be 10%. The reduction in capacity between the two methods is due to the 2D EDA tangent stiffness not accounting for the reduction in load from major-minor moment interaction. This shows that the tangent stiffness is a function of both axial load and moment.

Table 5. 3D frame comparison results

	Major axis frame max load (N)	Minor axis frame load (N)
3D EDA method	40700	19500
Lu (2009) EDA method	44500	21400

The 3D EDA frame does not currently have warping or torsion accounted for. By unlocking rotational freedoms in the 3D EDA method model it was seen that torsional effects occur after yielding. These torsional effects could be significant and further research is required to account for these effects.

## 4. CONCLUSIONS AND RECOMMENDATIONS

From the multiple analyses, answers to the key questions in section 1 were found to be:

- Residual stresses can be taken into account by using a parabolic residual stress profile on the section, with the max residual stress being equal to  $0.3f_y$ . Out of straightness can be taken into account through the use of a half sine wave.  $L/1000$  should be used for the maximum out-of-straightness in the minor axis while  $L/500$  should be used for the maximum out-of-straightness in the major axis (where  $L$  is the length of the column). Consideration of both of these effects accurately replicate the  $\alpha_B = 0$  New Zealand column design curve.

- b) From the analysis it was found that path dependency is a capacity reducing effect for steel columns. It was found that the capacity can differ by up to 18% for 1% monatomic drift depending on load path regime, with the worst case being when major and minor axis loads are applied at the same time.
- c) The difference between current New Zealand standards and 3D EDA is that the EDA method specifies the tangent stiffness as function of the moment, axial force and path dependency, where the New Zealand code uses magnification factors which are inaccurate for complex geometries.

In comparing results from the 3D frame, it was found that Lu (2009) 2D EDA method over predicted the capacity of a structure by 10% when compared to the 3D EDA method. This is due to the 2D EDA method only considering the tangent modulus as a function of axial load. This was shown through the smaller critical horizontal load generated through the 3D EDA method in the push over analysis.

#### 4.1. Further Development

Several assumptions were made for the 3D steel frame analysis:

- Lateral-torsional-buckling effects restrained
- Warping effects restrained
- Shear force effects ignored

In reality these effects also act significantly to reduce the capacity of the members under certain loading conditions. Therefore, it can be said that 3D EDA method is currently un-conservative.

For 3D EDA to consider all capacity reducing effects, the tangent stiffness needs to be specified as a function of moment in the member, axial force, torsion, shear and path dependency.

Further research needs to be conducted in order develop 3D EDA so these capacity reducing effects can be considered.

#### REFERENCES

- American Institute of Steel Construction. (2005). *Steel Construction Manual*
- Filippou, F. C. (2002). FEDEASLab, <http://FEDEASLab.berkeley.edu/>
- Lu, Y. C. (2009). *Extended Direct Analysis (EDA) of steel frames*
- Standards New Zealand. (2004). NZS 3404: Part1. *Steel structures standard*
- Ziemian, R, D & McGuire, W. (2000). MASTAN2, <http://www.mastan2.com/index.html>